Effects of anisotropy on optimal dense coding

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Abstract

We study optimal dense coding with thermal entangled states of a two-qubit anisotropic *XXZ* model and a Heisenberg model with Dzyaloshinski-Moriya (DM) interactions. The DM interaction is another kind of anisotropic antisymmetric exchange interaction. The effects of these two kinds of anisotropies on dense coding are studied in detail for both the antiferromagnetic and ferromagnetic cases. For the two models, we give the conditions that the parameters of the models have to satisfy for a valid dense coding. We also found that even though there is entanglement, it is unavailable for our optimal dense coding, which is the same as entanglement teleportation.

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I. INTRODUCTION

Entanglement is one of the most fascinating features of quantum mechanics and plays a central role in quantum information processing, such as quantum key distribution[1], quantum teleportation [2], dense coding [3], and so on. In the initial dense coding protocol [4], the sender can transmit two bits of classical information to the receiver by sending a single qubit if they share a two-qubit maximally entangled state (an Einstein-Podolsky-Rosen (EPR) state). Since then, many works on dense coding have been presented experimentally [5] or theoretically [6, 7, 8]. We know, in a general dense coding, the sender performs one of the local unitary transformations $U_i \in U(d)$ on d-dimensional quantum system to put the initially shared entangled state ρ in $\rho_i = (U_i \otimes I_d)\rho(U_i^\dagger \otimes I_d)$ with a priori probability $p_i (i = 0, 1, ..., i_{max})$, and then the sender sends off his quantum system to the receiver. Upon receiving this quantum system, the receiver performs a suitable measurement on ρ_i to extract the signal. The optimal amount of information that can be conveyed is known to be bounded from by the Holevo quantity[9]: $\chi = S(\overline{\rho}) - \sum_{i=0}^{i_{max}} p_i S(\rho_i)$, where $S(\rho)$ denotes the von Neumann entropy and $S(\overline{\rho}) = \sum_{i=0}^{i_{max}} p_i \rho_i$ is the average density matrix of the signal ensemble. Since the Holevo quantity is asymptotically achievable[10], one can use $\chi = S(\overline{\rho}) - \sum_{i=0}^{i_{max}} p_i S(\rho_i)$ as the definition of the capacity of dense coding. Moreover, the von Neumann entropy is invariant under unitary transformations, $S(\rho_i) = S(\rho)$. Therefore, the dense coding capacity can be rewritten as $\chi = S(\overline{\rho}) - S(\rho)$. The following problem is to find the optimal signal ensemble $\{\rho_i; p_i\}_{i=0}^{i_{max}}$ that maximizes χ. In Ref.[11], the author showed that the d^2 signal states ($i_{max} = d^2 - 1$) generated by mutually orthogonal unitary transformations with equal probabilities yield the maximum χ, which is called optimal dense coding, and considered the optimal dense coding when the shared entangled state was a general mixed one.

The quantum entanglement in solid state systems such as spin chains has been an important emerging field since the founding of thermal entanglement [12]. Spin chains are natural candidates for the realization of the entanglement compared with other physical systems. As the thermal fluctuation is introduced into the system, the state of a typical solid-state system at thermal equilibrium (temperature T) is $\rho(T) = e^{-\beta H}/Z$, where H is the Hamiltonian, $Z = tre^{-\beta H}$ is the partition function and $\beta = 1/(kT)$, where k is the Boltzman constant. For simplicity, we write $k = 1$. As $\rho(T)$ represents a thermal state, the entanglement in

the state is called the thermal entanglement. The study of thermal entanglement properties in Heisenberg systems has received a great deal of attention [13, 14, 15, 16, 17, 18]. Some authors have considered the availability of thermal entanglement. Quantum teleportation that uses the thermal entangled state as a channel has been proposed in Refs. [19, 20, 21].

In this paper, we study the optimal dense coding [11] using the thermal entangled states of a two-qubit anisotropic XXZ model and a Heisenberg model with DM interactions. We investigate the effects of two kinds of anisotropies on dense coding in detail for both the antiferromagnetic (AFM) and ferromagnetic (FM) cases and give the conditions that the parameters of the model have to satisfy for dense coding. The paper is organized as follows: in Sec. II, the optimal dense coding using the thermal entangled states of a two-qubit anisotropic *XXZ* model is investigated; we study the optimal dense coding using the thermal entangled state of a Heisenberg model with DM interactions in Sec. III and conclude in Sec. IV.

II. OPTIMAL DENSE CODING USING THERMAL STATES OF A TWO-QUBIT ANISOTROPIC *XXZ* CHAIN

We consider a two-qubit anisotropic *XXZ* Heisenberg model

$$H = \frac{J}{2}(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y} + \Delta\sigma_{1z}\sigma_{2z}) = J(\sigma_{1+}\sigma_{2-} + \sigma_{1-}\sigma_{2+}) + \frac{J\Delta}{2}\sigma_{1z}\sigma_{2z}, \tag{1}$$

where $\sigma_{j\alpha}$ $(j = 1, 2, \alpha = x, y, z)$ are the pauli matrices. J is the real coupling constant, $J > 0$ corresponding to the antiferromagnetic (AFM) case and $J < 0$ to the ferromagnetic (FM) case. The operators $\sigma_{j\pm} = (1/2)(\sigma_{jx} \pm i\sigma_{jy})$. Without loss of generality, we define $|0\rangle$ ($|1\rangle$) as the ground (excited) state of a two-level particle. The eigensystem of H is $H|00\rangle = \frac{J\Delta}{2}|00\rangle$, $H|\Psi^{\pm}\rangle = (-J\Delta/2 \pm J)|\Psi^{\pm}\rangle$ and $H|11\rangle = \frac{J\Delta}{2}|11\rangle$, where $|\Psi^{\pm}\rangle = (1/\sqrt{2})(|01\rangle \pm |10\rangle)$ are the two-qubit maximally entangled states (EPR states). The thermal state of the system at equilibrium (temperature T) is

$$\rho = \frac{1}{Z_1}[e^{-\beta\frac{J\Delta}{2}}|00\rangle\langle 00| + e^{-\beta(-\frac{J\Delta}{2}+J)}|\Psi^+\rangle\langle\Psi^+| + e^{-\beta(-\frac{J\Delta}{2}-J)}|\Psi^-\rangle\langle\Psi^-| + e^{-\beta\frac{J\Delta}{2}}|11\rangle\langle 11|], \tag{2}$$

where $Z_1 = 2\lambda e^{-J/2T}$ is the partition function and $\lambda = 1 + e^{J\Delta/T}\cosh[J/T]$. In Ref. [22], the concurrence [23] of the model is considered as a measure of thermal entanglement.

Now we carry out the optimal dense coding with the thermal entangled states of the two-qubit system as a channel. The set of mutually orthogonal unitary transformations [11]

of the optimal dense coding for two-qubit is

$$\begin{aligned} U_{00}|x\rangle &= |x\rangle, U_{10}|x\rangle = e^{\sqrt{-1}(2\pi/2)x}|x\rangle, \\ U_{01}|x\rangle &= |x+1(\mathtt{mod2})\rangle, U_{11}|x\rangle = e^{\sqrt{-1}(2\pi/2)x}|x+1(\mathtt{mod2})\rangle, \end{aligned} \quad (3)$$

where $|x\rangle$ is the single qubit computational basis ($|x\rangle = |0\rangle, |1\rangle$). The average state of the ensemble of signal states generated by the unitary transformations Eq. (3) is

$$\overline{\rho^*} = \frac{1}{4}\sum_{i=0}^{3}(U_i \otimes I_2)\rho(U_i^+ \otimes I_2), \quad (4)$$

where we have assumed $0 \rightarrow 00$; $1 \rightarrow 01$; $2 \rightarrow 10$; $3 \rightarrow 11$, and ρ is the thermal states of Eq. (2). Through straightforward algebra, we have

$$\overline{\rho^*} = \frac{1}{4}[|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|]. \quad (5)$$

After completing the set of mutually orthogonal unitary transformations, the maximum dense coding capacity χ can be written as

$$\chi = S(\overline{\rho^*}) - S(\rho) = 2 - S(\rho). \quad (6)$$

Here, $S(\rho)$ is the von Neumann entropy of the quantum state ρ. Thus, the value of the maximal dense coding capacity is

$$\chi = \frac{T\lambda(\ln[4] - 2\ln[\lambda]) + 2J\xi e^{\frac{J\Delta}{T}}}{T\lambda\ln[4]}, \quad (7)$$

where $\xi = \Delta\cosh[\frac{J}{T}] + \sinh[\frac{J}{T}]$.

It is found that $\chi(J, \Delta) = \chi(-J, -\Delta)$, which indicates that the maximal dense coding capacity χ satisfy $\chi_{AFM}(\Delta) = \chi_{FM}(-\Delta)$. The result is the same as the concurrence [22]. We give the numerical analysis of χ. In Fig. 1, the optimal dense coding capacity as a function of the coupling constant J and anisotropy Δ is plotted at a definite temperature. From the analytical point of view, in order to carry out the optimal dense coding successfully, the parameters of the model must satisfy

$$\chi > \log_2[2] = 1 \Leftrightarrow J\xi e^{\frac{J\Delta}{T}} > T\lambda\ln[\lambda]. \quad (8)$$

For different values of the anisotropy Δ and the coupling constant J, there must be a critical temperature $T_{critical}$, beyond which we cannot give an optimal dense coding with this two-qubit Heisenberg XXZ chain. In the following, we will investigate explicitly the effects of Δ and T on χ:

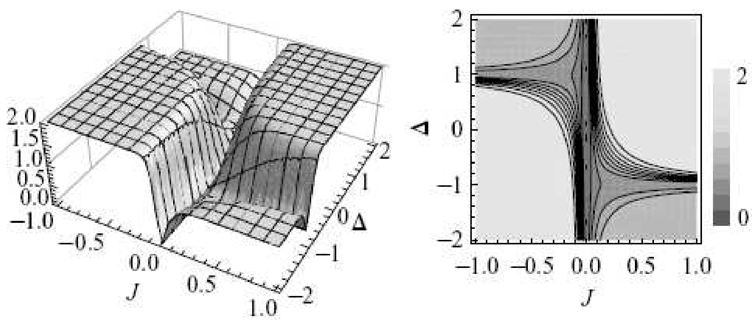

FIG. 1: (Color online) The optimal dense coding capacity χ versus coupling constant J and anisotropy Δ. We assume $T = 0.05$. The right panel is the contour.

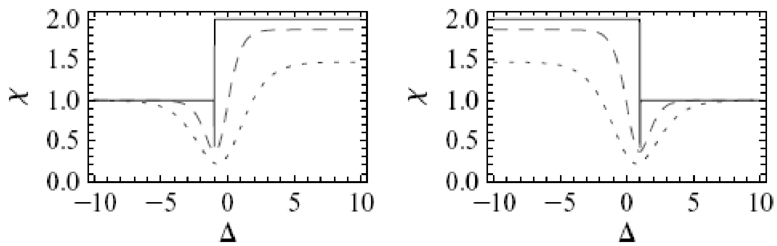

FIG. 2: (Color online) The optimal dense coding capacity χ versus anisotropy Δ. The left panel corresponds to an AFM case ($J = 1$) and the right panel corresponds to a FM one ($J = -1$). From top to bottom, the temperature is 0.005, 0.5, 1, respectively.

Case1: Anisotropy $\Delta = 0$. We find $\chi = 2$ when $T \to 0$, is always true regardless of AFM or FM case. The result is same when the channel is a two-qubit EPR state. It is because in this case, the thermal state is $|\Psi^-\rangle$ (for AFM) or $|\Psi^+\rangle$ (for FM), which are the EPR states, so the sender can transmit 2 bits classical information by sending 1 qubit.

Case2: Anisotropy $|\Delta| \gg 0$. We have

$$\begin{cases} \chi = \frac{1+T\ln[4]-T\ln[1+e^{2/T}]+\tanh[\frac{1}{T}]}{T\ln[2]}, & \text{if } \Delta \to +\infty; \\ \chi = 1, & \text{if } \Delta \to -\infty. \end{cases} \tag{9}$$

for $J = 1$,

$$\begin{cases} \chi = 1, & \text{if } \Delta \to +\infty; \\ \chi = \frac{1+T\ln[4]-T\ln[1+e^{2/T}]+\tanh[\frac{1}{T}]}{T\ln[2]}, & \text{if } \Delta \to -\infty. \end{cases} \tag{10}$$

for $J = -1$. These features can be manifested from Fig.2. For $J = 1$ and $\Delta \to -\infty$ or $J = -1$ and $\Delta \to +\infty$, we have $\chi = 1$, which means the quantum channel is not valid

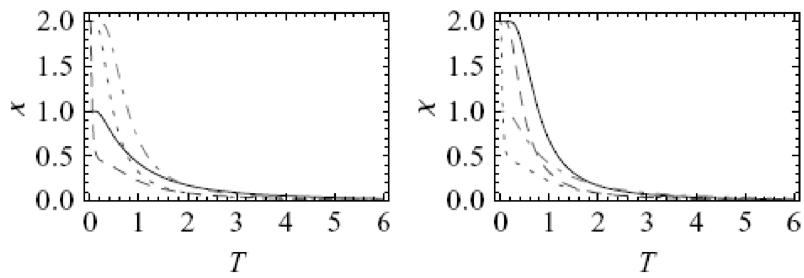

FIG. 3: (Color online) The optimal dense coding capacity χ versus temperature T. The left panel corresponds to an AFM case ($J = 1$) and the right panel corresponds to a FM one ($J = -1$). Left panel: Solid (Black) curve for $\Delta = -2$, Dashed (Blue) curve for $\Delta = -0.9$, Dotted (Red) curve for $\Delta = 0$, Dash-dotted (Green) curve for $\Delta = 1$. Right panel: Solid (Black) curve for $\Delta = -1$, Dashed (Blue) curve for $\Delta = 0$, Dotted (Red) curve for $\Delta = 0.9$, Dash-dotted (Green) curve for $\Delta = 2$.

for optimal dense coding, for now the thermal state is $\frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$, which is a superposition of two product states. But for $J = 1$ and $\Delta \to +\infty$ or $J = -1$ and $\Delta \to -\infty$, the value of the maximal dense coding capacity is the same and depends on the temperature. In these two cases, the thermal state is

$$\rho = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm\tanh[\frac{1}{T}] & 0 \\ 0 & \pm\tanh[\frac{1}{T}] & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{11}$$

where '+' corresponds $J = -1$ and $\Delta \to -\infty$, '−' corresponds $J = 1$ and $\Delta \to +\infty$. The concurrence [23] of Eq.(11) is $C = \tanh[\frac{1}{T}]$, which means Eq. (11) is an entangled state for a finite temperature. In order to get a valid optimal dense coding when anisotropy $|\Delta| \gg 0$, here we must have $\frac{1+T\ln[4]-T\ln[1+e^{2/T}]+\tanh[\frac{1}{T}]}{T\ln[2]} > 1 \Leftrightarrow \frac{1+\tanh[1/T]}{\ln[(1+e^{2/T})/2]} > T$. This can be held for any temperature. We can also see this from Fig.2. Moreover, since $\chi(J = 1, \Delta = -1) = \chi(J = -1, \Delta = 1) = \ln[4/3]/\ln[2] \approx 0.415037$ when $T \to 0$, the value will be smaller as the temperature increases. This can be understood since the concurrence of this model is zero when $\Delta = -1$ for AFM and $\Delta = 1$ for FM. Therefore, χ has an abrupt transition at the two points.

Case3: For a finite anisotropy and temperature. we plot the optimal dense coding capacity

χ as a function of the temperature T for four different values of anisotropy in Fig.3. From the figure, when $J = 1$, if $\Delta < -1$, regardless of what temperature, χ is always less than 1 and the thermal entangled states are not valid for optimal dense coding. This can be explained since the concurrence of this model $C_{AFM} = 0$ for $\Delta < -1$. Moreover, we can see with the increase of the Δ, the area of T that the optimal dense coding is feasible becomes wider. Accordingly, for $J = -1$, we must have $\Delta < 1$ in order to make $\chi > 1$ at some temperature. This is easily understood because $C_{FM} = 0$ for $\Delta > 1$. But the area of T that the optimal dense coding is feasible becomes more narrow.

III. THE EFFECTS OF DM INTERACTION ON OPTIMAL DENSE CODING

Another kind of anisotropy that we investigate is the DM anisotropic antisymmetric interaction which arises from spin-orbit coupling [24, 25]. Now we consider the Heisenberg model with DM interaction

$$H_{DM} = \frac{J}{2}[(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y} + \sigma_{1z}\sigma_{2z}) + \overrightarrow{D} \cdot (\overrightarrow{\sigma_1} \times \overrightarrow{\sigma_2})], \tag{12}$$

here $\overrightarrow{D}$ is the DM vector coupling. For simplicity, we choose $\overrightarrow{D} = D\overrightarrow{z}$, and the Hamiltonian H_{DM} becomes

$$\begin{aligned} H_{DM} &= \frac{J}{2}[\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y} + \sigma_{1z}\sigma_{2z} + D(\sigma_{1x}\sigma_{2y} - \sigma_{1y}\sigma_{2x})] \\ &= J[(1+iD)\sigma_{1+}\sigma_{2-} + (1-iD)\sigma_{1-}\sigma_{2+} + \frac{J}{2}\sigma_{1z}\sigma_{2z}]. \end{aligned} \tag{13}$$

We notice that $H_{DM}(D = 0) = H(\Delta = 1)$. The eigenvalues and eigenvectors of H_{DM} are $H_{DM}|00\rangle = \frac{J}{2}|00\rangle$, $H_{DM}|11\rangle = \frac{J}{2}|11\rangle$, $H_{DM}|\pm\rangle = (\pm J\sqrt{1+D^2} - \frac{J}{2})|\pm\rangle$, with $|\pm\rangle = (1/\sqrt{2})(|01\rangle \pm e^{i\theta}|10\rangle)$ and $\theta = \arctan D$.

As the thermal fluctuation is introduced into the system, in the standard basis $\{|11\rangle, |10\rangle, |01\rangle, |00\rangle\}$, the state can be expressed as

$$\rho_{DM} = \frac{1}{Z_2}\begin{pmatrix} e^{-\beta J/2} & 0 & 0 & 0 \\ 0 & \frac{1}{2}e^{\frac{1}{2}\beta(J-\delta)}(1+e^{\beta\delta}) & \frac{1}{2}e^{i\theta}e^{\frac{1}{2}\beta(J-\delta)}(1-e^{\beta\delta}) & 0 \\ 0 & \frac{1}{2}e^{-i\theta}e^{\frac{1}{2}\beta(J-\delta)}(1-e^{\beta\delta}) & \frac{1}{2}e^{\frac{1}{2}\beta(J-\delta)}(1+e^{\beta\delta}) & 0 \\ 0 & 0 & 0 & e^{-\beta J/2} \end{pmatrix}, \tag{14}$$

where $Z_2 = 2\eta e^{-\frac{J}{2T}}$, $\eta = 1 + e^{J/T}\cosh[\delta/(2T)]$ and $\delta = 2J\sqrt{1+D^2}$. The entanglement of this model has been studied [21] by means of concurrence. Through Eq.(3) and Eq.(4), we

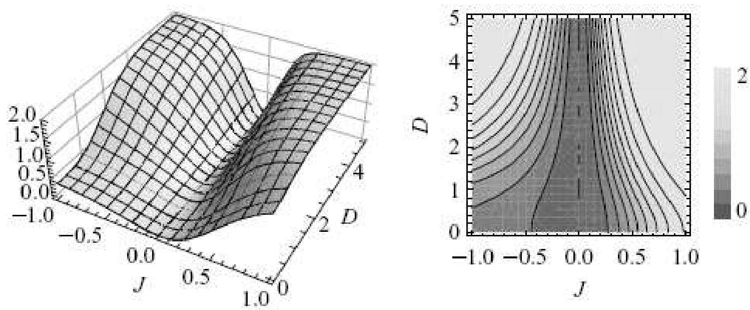

FIG. 4: (Color online) The optimal dense coding capacity χ versus coupling constant J and DM interaction D. We assume $T = 0.5$. The right panel is the contour.

have also

$$\overline{\rho^*}_{DM} = \frac{1}{4}[|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|]. \tag{15}$$

Similarly, after a straight calculation, the value of the maximal dense coding capacity is given by

$$\chi = \frac{T\eta(\ln[4] - 2\ln[\eta]) + \zeta e^{\frac{J}{T}}}{T\eta \ln[4]}, \tag{16}$$

where $\zeta = 2J\cosh[\delta/(2T)] + \delta \sinh[\delta/(2T)]]$. In order to carry out the optimal dense coding successfully, the parameters of the model must satisfy $\chi > \log_2[2] = 1 \Leftrightarrow \zeta e^{\frac{J}{T}} > 2T\eta\ln[\eta]$, which can be returned to Eq.(8) where $\Delta = 1$ for a vanishing DM interaction.

We give a plot of optimal dense coding capacity as a function of DM interaction and spin coupling constant at a definite temperature in Fig.4. The variation of χ with D is very similar to that of concurrence for the AFM case. But for the FM case, the behavior of χ is different from that of concurrence. These results can be found by comparing Fig.4 with Fig.1 in Ref. [21]. Next, we consider the effects of DM interaction on dense coding capacity.

Case1: DM interaction $D \gg 0$. The optimal dense coding capacity is always equal to $log_2[4] = 2$ for the AFM and FM cases since the thermal is $\frac{1}{2}[|01\rangle\langle 01| \mp i|01\rangle\langle 10| \pm i|10\rangle\langle 01| + |10\rangle\langle 10|]$, which is a EPR type state and its concurrence is 1.

Case2: For a finite DM interaction and temperature. The variation of χ with D for $J = 1$ and $J = -1$ is plotted in Fig.5. As temperature increases, the useful area of D for optimal dense coding becomes narrow whether $J > 0$ or $J < 0$. Although there exists some D for the thermal state that is not valid for optimal dense coding, for these D the concurrence of the model is not zero. Therefore, even though there is entanglement, it is

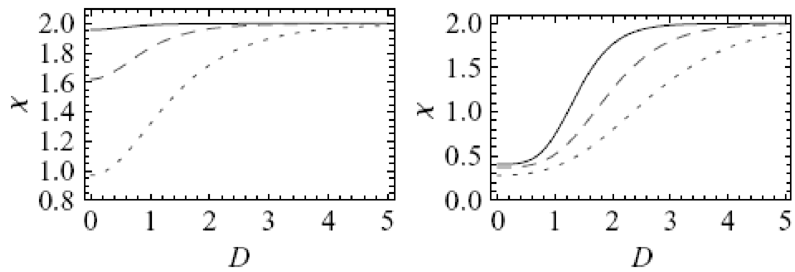

FIG. 5: (Color online) The optimal dense coding capacity χ versus DM interaction D. The left panel corresponds to an AFM case ($J = 1$) and the right panel corresponds to a FM case ($J = -1$). From top to bottom, the temperature is 0.3, 0.5, 0.8, respectively.

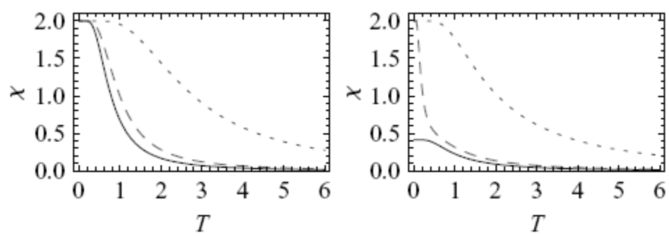

FIG. 6: (Color online) The optimal dense coding capacity χ versus the temperature. The left panel corresponds to an AFM case ($J = 1$) and the right panel corresponds to a FM case ($J = -1$). From top to bottom, the DM interaction is 5, 1, 0, respectively.

unavailable for our optimal dense coding, which is the same as entanglement teleportation. Comparing the left with the right, DM interaction must be stronger for FM in order to make $\chi > 1$ at the same temperature for AFM. Moreover, $D = 0$, χ is always less than 2 for the FM case no matter what temperature is, which can be easily understood since the entanglement is zero. In Fig.6, the optimal dense coding capacity as a function of the temperature is plotted for different DM interactions. The critical value of T when the thermal entangled states is valid for optimal dense coding ($\chi > 1$) for $D = 5$ is larger than that for $D = 1$. At zero temperature, regardless of the DM interaction, $\chi = 2$ for the AFM because the state is $|-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - e^{i\theta}|10\rangle)$, which is a EPR type state. However, the thermal state is uncertain for the FM case at zero temperature. For nonzero D, the state is $|+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + e^{i\theta}|10\rangle)$, so $\chi(D \neq 0) = 2$. At zero temperature, $\chi(D = 0) < 1$ because the state is $\frac{1}{6}[2|00\rangle\langle 00| + |01\rangle\langle 01| + i|01\rangle\langle 10| - i|10\rangle\langle 01| + |10\rangle\langle 10| + 2|11\rangle\langle 11|]$, which is not an entangled state.

IV. CONCLUSIONS

In conclusion, we studied analytically the effects of two kinds of anisotropy on the optimal dense coding in the anisotropic *XXZ* model and the Heisenberg model with DM interaction. We demonstrated that whether the optimal dense coding is valid or not depends on both the anisotropic parameters and the sign of exchange constants J. The conditions for a valid optimal dense coding have been given. For the AFM *XXZ* model, anisotropy must be larger than -1 and the critical temperature above which the optimal dense coding capacity $\chi < 1$ will increase as anisotropy increases. But anisotropy must be less than 1 and the critical temperature will decrease with the increasing of anisotropy for the FM case. The dependence trends of optimal dense coding capacity on DM interaction are the same for both the AFM and FM Heisenberg model with DM interaction, and the relatively stronger DM interaction will be helpful for optimal dense coding. We also found that even though there is entanglement, it is unavailable for our optimal dense coding, which is the same as entanglement teleportation.

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[1] C. H. Bennett, D. P. Divincenze, Nature (London) 404, 247 (2000).

[2] A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991).

[3] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993) .

[4] C. H. Bennett, S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992).

[5] K. Mattle, H. Weinfurter, P. G. Kwiat, A. Zeilinger, Phys. Rev. Lett. 76, 4656 (1996).

[6] A. Bareno, A. Ekert, J. Mod. Opt. 42, 1253 (1995).

[7] S. L. Braunstein, H. J. Kimble, Phys. Rev. A 61, 042302 (2000).

[8] S. Bose, M. B. Plenio, V. Vedral, J. Mod. Opt. 47, 291 (2000).

[9] A. S. Holevo, Probl. Inf. Transm., **9**(3), 177-183 (1973); A. S. Holevo, Probl. Inf. Transm., **15**(4), 247-253 (1979).

[10] A. S. Holevo, IEEE Trans. Inf. Theory, **44**, 269 (1998); B. Schumacher and M. D. Westmoreland, Phys. Rev. A **56**, 131 (1997).

[11] T. Hiroshima, J. Phys. A **34**, 6907 (2001).

[12] M. A. Nielsen, Ph.D thesis, University of New Mexico, 1998, quant-ph/0011036.

[13] X. Wang, Phys. Rev. A **64**, 012313 (2001).

[14] D. V. Khveshchenko, Phys. Rev. B **68**, 193307 (2003).

[15] U. Glaser, H. Büttner, H. Fehske, Phys. Rev. A **68**, 032318 (2003).

[16] G. Vidal, J. I. Latorre, E. Rico, A. Kitaev, Phys. Rev. Lett. **90**, 227902 (2003).

[17] G. K. Brennen, S. S. Bullock, Phys. Rev. A **70**, 52303 (2004).

[18] F. Verstraete, M. Popp, J. I. Cirac, Phys. Rev. Lett. **92**, 027901 (2004).

[19] Y. Yeo, Phys. Rev. A **66**, 062312 (2002); Y. Yeo, T. Liu, Y.E. Lu, Q.Z. Yang, J. Phys. A **38**, 3235 (2005).

[20] Z. C. Kao, J. Ng, Y. Yeo, Phys. Rev. A **72**, 062302 (2005).

[21] G. F. Zhang, Phys. Rev. A **75**, 034304 (2007).

[22] X. G. Wang, Phys. Lett. A **281**, 101 (2001).

[23] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).

[24] I. Dzyaloshinskii, J. Phys. Chem. Solids **4**, 241 (1958).

[25] T. Moriya, Phys. Rev. Lett. **4**, 228 (1960); T. Moriya, Phys. Rev. **117**, 635 (1960); T. Moriya, Phys. Rev. **120**, 91 (1960).